H^2- and H^∞-Design Tools for Linear Time-Invariant Systems

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Abstract

Recent advances in optimal control have brought design techniques based on optimization of H^2 and H^∞ norm criteria, closer to be attractive alternatives to single-loop design methods for linear time-invariant systems. Significant steps forward in this technology are the deeper understanding of performance and robustness issues of these design procedures and means to perform design trade-offs. However acceptance of the technology has been hindered by the lack of convenient design tools to exercise these powerful multivariable techniques, while still allowing single-loop design formulation. Presented in this paper is a unique computer tool for designing arbitrary low-order linear time-invariant controllers that encompasses both performance and robustness issues via the familiar H^2 and H^∞ norm optimization. Application to disturbance rejection design for a commercial transport is demonstrated.

I. Introduction

Past three decades have laid a foundation on the theory of optimal control. Issues have been actively pursued in algorithms for numerical solution of optimum designs, feedback properties of optimal linear feedback (and feedforward) controllers and associated theoretical results of existence and uniqueness. Filtering of these wealth of technology down to current practitioners have been agonizingly slow. Demonstration and acceptance of these design techniques in typical flight systems such as SAS (stability augmentation systems), manual controls and autopilot designs, are almost non-existent. Hindrances in this effort are related to concerns raised in the following areas: design simplicity, ease-of-modification during flight-test and incorporation of designers' intuition and experiences in these "optimum" systems. Presented in this paper is the development of a design tool that covers much of the advances in multivariable controls and its potential application to flight controls.

II. Background and Motivation

Historically multivariable controls have been extensively developed based on optimal control of linear time-invariant systems. Class of design problems addressed in the past are optimal linear regulator using full-state feedback or estimate-state output feedback. Research efforts to extend the usefulness of multivariable control designs within the reach of experienced control designers are concentrated in the following areas:

- Measures of design robustness in the presence of modeling uncertainties[†1-21],

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† Numbers indicate references.

- Synthesis methods to achieve trade-offs in performance and robustness[7,22-35],
- Model and controller order reduction[36-39],
- Direct synthesis of reduced-order controllers of given structure to achieve trade-off in closed-loop performance and robustness, and at the same time facilitate design integration over different operating conditions[23].

A well-known property of feedback concept is its ability to regulate in the presence of plant uncertainties. Measures of robustness are traditionally based on loop stability margins[40] (i.e. phase and gain margins of individual control loops while maintaining other loops closed at nominal gains). These single-loop robustness tests provide useful design criteria for the evaluation of current flight control systems. Recent development in robustness analysis techniques allow designers to examine design sensitivity in the multiloop and multivariable settings. To detect conditions for design sensitivity, one makes use of the singular values of loop return-difference matrix at appropriate plant input/output locations. In addition μ-measure is defined to characterize design robustness to uncertainties that are expressed in terms of plant parameter variations or those that have a predetermined structure. The latter robustness measure, μ-measure, is difficult to evaluate exactly; but it provides the most accurate description of design robustness in the presence of structured uncertainties. Current research direction is to devise numerical schemes that approximate closely[12] (i.e. providing "least" conservative upper and lower bounds) or, exactly calculate the μ-function for some specific types of structured uncertainties[13-21]. Synthesis methods to improve design robustness based on a general μ-measure are not available.

Guaranteed robustness of (-6dB,∞) in gain margins and (-60°,60°) in phase margins from optimal linear regulator have motivated researchers in developing design procedures to retain or recover these robustness properties for state-estimate feedback controllers[7] (i.e. as in optimal linear quadratic gaussian (LQG) designs). Fundamental understanding in the robustness recovery process resulted in design methods classified under the category of LQG/LTR where, for example, loop properties of a full-state feedback regulator are asymptotically recovered with appropriate selection of process noise models[41], or in the framework of "asymptotic" pole assignment[42]. One possible drawback of these procedures is the tendency of having unnecessarily high gains in the estimator design during the loop transfer recovery.

Controllers obtained from a LQG/LTR design procedure are usually of high order (i.e. order of the controlled plant model augmented with models for actuation, sensor and design-shaping filter dynamics). Implementation of these controllers in digital flight processors may be feasible with anticipated advances in computing technology; but will undoubtedly be challenging from the point of view of design traceability, reliability and maintainability. However it is often possible to reduce the controller order using standard model reduction techniques such as modal residualization[43], balanced truncation[44] and optimal Hankel norm approximation[37]. Careful considerations must be made especially in the reduction of controllers so that closed-loop stability, performance and robustness are preserved. Frequency-weighted reduction schemes have been developed to address these issues with some success[39].

A remaining problem is the final integration of "suboptimal" reduced-order controllers to operate over a wide range of flight conditions. This is usually achieved in an ad-hoc manner (e.g. curve-fitting gain parameters optimized at individual flight conditions as a function of some physical quantities such as calibrated airspeed, aircraft weight, aircraft center of gravity and dynamic pressure).

With some of the above outstanding issues unresolved, it is evident that wide acceptance of these design techniques has been difficult. Additional reason behind this difficulty in technology transfer is the lack of intuitiveness in the design approach to handle low-order controllers of given (i.e. predetermined) structures (e.g. washout filter in the yaw damper design of a lateral control system for turn-coordination, simple a-priori gain scheduling according to physical quantities such as aircraft weights, dynamic pressure and calibrated airspeed, synthesis of dedicated structural filters for control of lightly passively damped elastic modes, etc...).

Research in direct synthesis of reduced-order controllers for multivariable controls are being actively pursued and have resulted in some promising design algorithms both in the continuous-time[23,32] and discrete-time[45,46] domains. Although theoretical development of the design techniques have made significant strive, insights in applying them to the synthesis of practical flight control systems are yet to be established. To facilitate the evaluation and technology transfer of multivariable control design concepts, there is a need for an efficient and versatile design tool that is able to resolve the above issues related to design implementation, performance, robustness and integration (i.e. gain schedule) over a wide range of operating conditions.

III. Design Tool Development

The objective of the design tool is to provide a unified framework for applying recent advances in robust multivariable controls to flight systems. To achieve this goal, development of efficient and versatile computational algorithms is needed. Scope of the design concept and procedure will hopefully enable and motivate experienced designers to appreciate the importance and value of multivariable controls. Steps taken to accomplish the stated objectives are as follows:

- Formulation of control design problems and solution algorithms for the synthesis of robust low-order controllers,
- Implementation of design algorithms in useful CAD tools for ease in obtaining design solutions to a wide class of linear feedback/feedforward controllers,
- Ability to formulate other design specifications using linear and nonlinear equality and inequality constraints.

Control Design Problem: Multivariable controls have primarily focused on applying optimization to the design of control systems. Extensive work conducted to-date are on control of linear time-invariant systems. The problem is the synthesis of linear controllers that meet specific closed-loop performance and robustness over a range of linearized plant conditions. Surprisingly this problem is identical to the one that experienced designers have to confront in their daily design work where traditional single-input single-output (SISO) methods prevail. Inadequacies of these SISO design techniques are well-known: neglect of cross-coupling effects, difficulty to satisfy multiple design requirements, highly dependent on the designers' experience, trial-and-error. On the other hand, advantages behind SISO design procedures are: the simplicity of its final controller structure, the ease-of-incorporating designers' experience into the design process and design flexibility for post-flight test modification. A useful design tool would combine these existing SISO design features into multivariable control synthesis.

Design Procedure Based on H^2 and H^∞-Optimization : Design methods for multivariable controls can be categorized into two general classes depending on whether the control-laws are synthesized based on minimizing a performance measure while satisfying other design constraints, or just simply meeting design constraints (e.g. eigenstructure assignment).

In the category of performance-oriented methods, control algorithms are generally developed from optimization of some weighted norms of the plant outputs and control inputs in a closed-loop system subject to deterministic or stochastic disturbances. Two commonly used measures are H^2 and H^∞-norms with interpretation in both frequency and time-domains. Until recently, and mostly for mathematical convenience, majority of feedback and feedforward control-law synthesis are based on H^2-norm. Associated design schemes are classified under the following methods: linear quadratic regulator (LQR), linear quadratic gaussian (LQG) design and, linear quadratic gaussian design with loop transfer recovery (LQG/LTR).

Over the past decade, practitioners of these techniques have gathered valuable experiences in applying these techniques to flight controls. Iterative procedures have been developed to achieve trade-offs between performance and control bandwidths[54,55]. However implementation of these designs remains an area of concern and need further research development. Often these designs with full-state feedback structure are implemented using a state estimator or observer. Procedures to obtain design robustness in state-estimate feedback are done through the mechanism of Riccati equation[41] or eigenvalue placement[42] starting from sufficient conditions for loop transfer recovery.

These procedures offer valuable insights into the design feasibility based on requirements of closed-loop stability, performance and robustness. Unfortunately, difficulty in extending these results to encompass traditional design philosophy (e.g. output feedback, low-order controller with structure intuitive to designers, gain scheduling,...) remains. Attempts to fit these multiloop high-order controllers into low-order and conceptually simple designs using, for example, controller order reduction are not trivial and have been partially successful[39,43]. This remains to be an area of continued research interests.

A completely different, direct and practical approach to multivariable controls would be via the route of parameter optimization. The control design procedure described in this paper is one of such methods. It is based on the optimization of an objective function using <u>any</u> pre-defined controller structure and subject to additional linear and nonlinear design constraints[56]. The formulation allows direct intervention of control designers through the set-up of the design objective function, the controller structure and constraints on closed-loop stability, performance and robustness to plant uncertainties. This design concept was originally developed in reference 23 for linear time-invariant systems using objective function based on H^2-norm. The design algorithm was efficiently implemented into the computer-aided-design package SANDY. Evaluation of the objective function and its gradients with respect to the controller parameters are performed analytically for a diagonalizable closed-loop system. Subsequent improvement have been made in the area of numerical optimization (e.g. found in the constrained optimization code NPSOL[56]), and in the development of typical constraints encountered in flight control systems such as closed-loop damping, covariance bounds on output and control variables. Encouraging results have been obtained in a variety of control design applications[22,26-28,31,48].

Reference 23 has also demonstrated the early application of such a technique in simple design situations. Later applications have been in the design of missile autopilot[27], design of a reliable stability and command augmentation system for a commercial transport[22,26], design of an improved lateral ride quality control system[48,49]. Usefulness of such a design tool has been further investigated in the control of a remotely-piloted vehicle[50,51] and nonrigid manipulators[52,53]. Reference 53 actually applied and verified in experiments results achieved using the design algorithm[23] to the synthesis of robust compensators for flexible structures with uncertain parameters.

This research has led to the development of a unified multivariable control design concept that addresses virtually all flight control design problems such as stability augmentation systems, gust load alleviation, manual and automatic control modes. Typical flight control systems can be formulated exactly in the same situation as designers would when conducting designs using single-loop frequency-domain techniques. However, in the design solution, multivariable control methods based on H^2 and H^∞-optimization will be used instead to derive the appropriate design gains. Section V illustrates briefly the design philosophy in the synthesis of a longitudinal control system of a commercial transport.

With this unique design concept developed for solution of optimal H^2-norm type of problems, the work is later extended to address control issues related to H^∞-norm[47]. The overall scope is to provide a unified design algorithm for low-order controller synthesis that utilizes criteria based on both H^2 and H^∞-norms[57,58]. To achieve this objective an efficient numerical algorithm has been developed to solve the following optimal control problems:

(a) Mixed H^2 and H^∞-Design Objective: Synthesis of feedback/feedforward controllers of fixed (i.e. arbitrary) order and structure is based on the minimization of the objective function J given by

$$J = \sum_{i=1}^{N_p} \left\{ W_\infty^i \left\| \begin{matrix} Q^{i\,1/2} H_{zw}^i(j\omega) \\ R^{i\,1/2} H_{uw}^i(j\omega) \end{matrix} \right\|_\infty^2 + W_2^i \left\| \begin{matrix} Q^{i\,1/2} H_{zw}^i(j\omega) \\ R^{i\,1/2} H_{uw}^i(j\omega) \end{matrix} \right\|_2^2 \right\} \tag{1.a}$$

or

$$J = \lim_{t_f \to \infty} \sum_{i=1}^{N_p} \left\{ W_\infty^i \sup_{w^i(t)} \frac{\int_0^{t_f} [z^T(t) Q^i z(t) + u^T(t) R^i u(t)] dt}{\int_0^{t_f} w^{i\,T}(t) w^i(t)] dt} + W_2^i\, E[z^T(t_f) Q^i z(t_f) + u^T(t_f) R^i u(t_f)] \right\} \tag{1.b}$$

Note that $\mathbf{H}_{zw}(s)$ and $\mathbf{H}_{uw}(s)$ are the transfer matrices between the disturbance inputs $\mathbf{w}(s)$ and the performance outputs $\mathbf{z}(s)$ and controls $\mathbf{u}(s)$ of the closed-loop system respectively.

This formulation covers design criteria that are expressible either in terms of H^2-norm or H^∞-norm, or a combination of the two. Another feature of this set-up is its ability to address design robustness to plant uncertainties (e.g. structured and unstructured uncertainties at both the plant inputs and plant outputs, plant parameter variations) through the use of aggregated closed-loop responses over a set of plant conditions, signified by the summation index i ($i=1,N_p$) and N_p is the total number of design conditions. An objective function that spans multiple plant conditions further provides a means to establish gain scheduling across the entire design envelope.

Reference 23 demonstrated the usefulness of this design formulation in controlling an 8^{th}-order flexible mechanical system under a non-collocated sensor/control configuration. A second-order controller has been designed that is robust to large variation in moment of inertia of one of the disks in a four-disk system[23,53]. The resulting robust controller turns out to be non-minimum phase. This result agrees with the SISO control synthesis procedure[59] for active vibration control.

The design algorithms for evaluating both H^2 and H^∞-norms use an equivalent time-domain characterization. The equivalence is established using the familiar Parseval theorem[60].

(b) H^2 Design Objective with H^∞-Bound Constraints: Alternatively one can define the control problem being the minimization of an objective function J based on H^2-norm,

$$J = \sum_{i=1}^{N_p} W_2^i \left\| \begin{matrix} Q^{i\,1/2} H_{zw}^i(j\omega) \\ R^{i\,1/2} H_{uw}^i(j\omega) \end{matrix} \right\|_2^2 \tag{2}$$

subject to additional constraints

$$\left\| \begin{matrix} Q^{i\,1/2} H^{i}_{zw}(j\omega) \\ R^{i\,1/2} H^{i}_{uw}(j\omega) \end{matrix} \right\|_{\infty} \leq \gamma_i \tag{3}$$

for some positive scalar γ_i ($i=1,N_p$). Recent work in H^∞-optimization[33] follow similar past development in optimal control for H^2-norm problems to a single plant model. Algorithms have been developed for state-feedback and output-feedback controllers (of the same order as the plant model) that satisfy given H^∞-bounds (e.g. equation (3)). These methods can be applied iteratively to yield solution of an H^∞-optimal control problem.

The resulting LQG-like controllers that are solutions to the H^∞-optimal control problems suffer the same drawbacks associated with traditional LQG controllers. Moreover, solutions of low-order controllers (i.e. strictly less than the order of the plant model) for H^∞-optimization are still not available. Our method provides a convenient framework for H^∞-optimization using the early design concept developed in reference 23 for low-order controllers. The outcome is a unified design procedure that allows control practitioners to examine requirements based on current findings in H^∞-bounds or other related measures (e.g. μ-measure, worst-case perturbations in parametric uncertainties) for performance and robustness.

Finite-Time Quadratic Performance Index : One unique feature of the design algorithms for H^2 and H^∞-norm calculation is the usage of a finite-time quadratic performance index. The objective function $J(t_f)$ (with a finite terminal time t_f) for both L^2 and L^∞ norms is given by the following equations,

- For H^2-norm :
  - Random Initial Conditions:

$$J(t_f) = \sum_{i=1}^{N_p} w_{p_i} \int_0^{t_f} e^{2\alpha t}\, E[z^T(t)Q^i z(t)+u^T(t)R^i u(t)]dt \leq J(t_f \rightarrow \infty) \tag{4.a}$$

  - Random Forcing Inputs:

$$J(t_f) = \sum_{i=1}^{N_p} w_{p_i}\, E_\alpha[z^T(t_f)Q^i z(t_f)+u^T(t_f)R^i u(t_f)] \leq J(t_f \rightarrow \infty) \tag{4.b}$$

or

$$J(t_f) = \frac{1}{t_f}\sum_{i=1}^{N_p} w_{p_i} \int_0^{t_f} e^{2\alpha t}\, E[z^T(t)Q^i z(t)+u^T(t)R^i u(t)]dt \leq J(t_f \rightarrow \infty) \tag{4.c}$$

- For H^∞-norm:

$$J(t_f) = \sum_{i=1}^{N_p} w_{p_i} \frac{\displaystyle\int_0^{t_f} [z^T(t)Q^i z(t)+u^T(t)R^i u(t)]dt}{\displaystyle\int_0^{t_f} w^{i\,T}(t)w^i(t)dt} \tag{5}$$

with $w^i(t)=w_o^i \exp(j\omega_o^i t)$ where w_o^i and ω_o^i are respectively the direction vector and frequency of the "worst-case" inputs $w(t)$ in the H^∞-norm evaluation at the i^{th} plant condition.

The formulation based on a finite-time horizon provides not only appropriate lower bounds to these exact norms, but also an indication on internal stability for disturbable and detectable systems. It is well-known that, if treated entirely in frequency domain, synthesis procedure that minimizes either the L^2 or L^∞-norms of plant outputs would not necessarily guarantee closed-loop stability. With the proposed formulation, if an optimum solution exists from the minimization of $J(t_f)$ for sufficiently large t_f, then closed-loop stability will be achieved for a detectable, disturbable and stabilizable system.

Another design concern in multivariable controls is the effect of input directionality[61] upon the closed-loop performance. Sensitivity of closed-loop responses to command inputs in the presence of plant uncertainties is often neglected or not explicitly defined in design techniques such as LQ, LQG, LQG/LTR. The design procedure described herein is based on parameter optimization of design objective that incorporates directly responses to specific commands. In this manner, effects of input directionality are obviously captured in the design objective through appropriately chosen input directions and with the usage of multiple plant conditions. It is envisioned that the ill-conditioned problem[61] of multivariable controls would no longer be a design issue.

IV. Design Tool Implementation (SANDY)

Figure 1 shows a schematics of the CAD design tool SANDY. The design tool is innovative and will serve a useful medium for the introduction of multivariable control concepts to a vast number of traditional control designers.

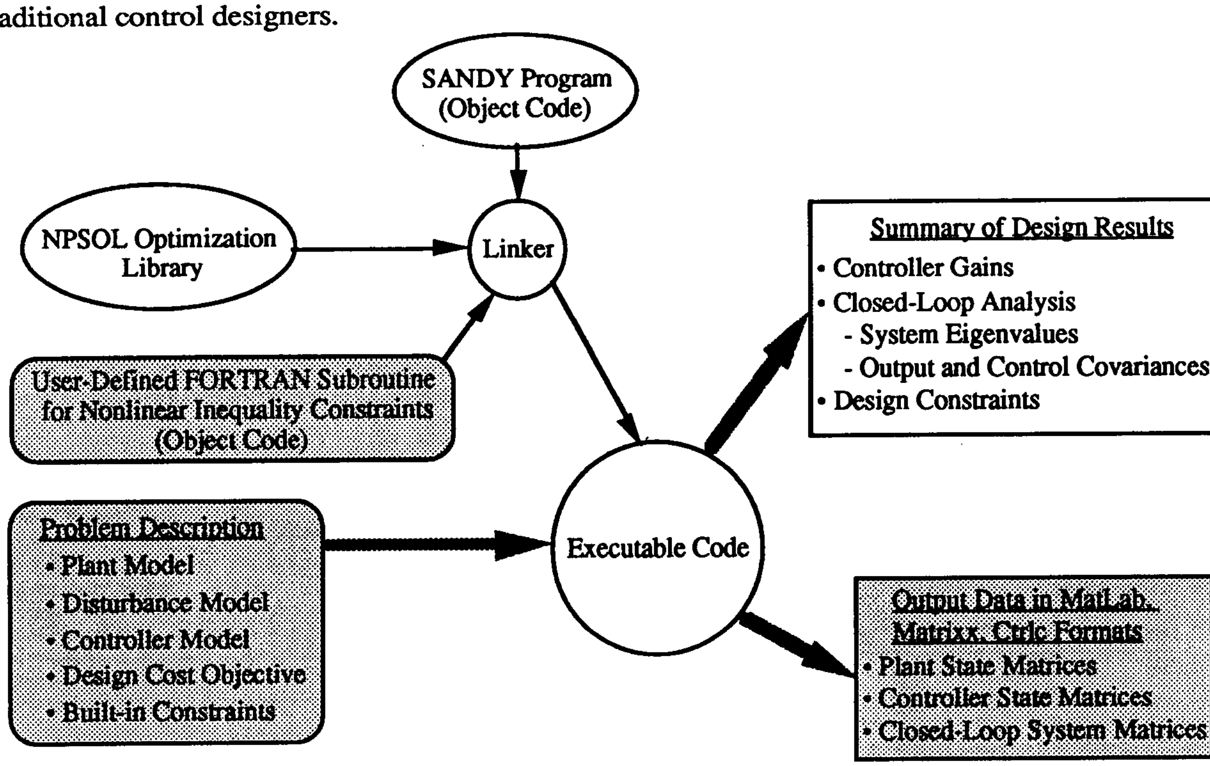

Figure 1 Schematics of the Design Tool Implementation SANDY

A procedure is set up to link the design code SANDY with the optimization library NPSOL and any user-defined Fortran subroutines defining nonlinear inequality constraints for the control design problem. This capability provides great flexibility to incorporate any additional design specifications to the problem without affecting the core program. An executable code is then generated to run the design optimization. This version of the executable code can be run repetitively without the need to relink when the designers switch between design conditions, alter the design parameters while keeping the same design constraints as defined in the user-defined Fortran subroutines.

Characteristics of the disturbance model, weighting parameters in the design objective, selection of the controller design parameters and options in built-in design constraints are entered in one single data file. State matrices for the plant models and the controller model are defined in separate data files. Printout of the design results will be provided at the end of the program execution. Future development will include the generation of design data files for the plant models, optimized controller model and the respective closed-loop systems in compatible formats for the analysis packages such as Matlab, Matrixx and Ctrlc.

V. Design Example

Preliminary development of the above design algorithm for mixed H^2 and H^∞-optimization has been applied to the disturbance rejection problem for a B767 aircraft (Figure 2). The design objective is to synthesize a low-order feedback controller that minimizes the aircraft normal acceleration $n_{zcg}(t)$ responses to vertical gust turbulence w(t). The performance relates to both peak responses (i.e. worst-case) and mean-square responses to Dryden spectra.

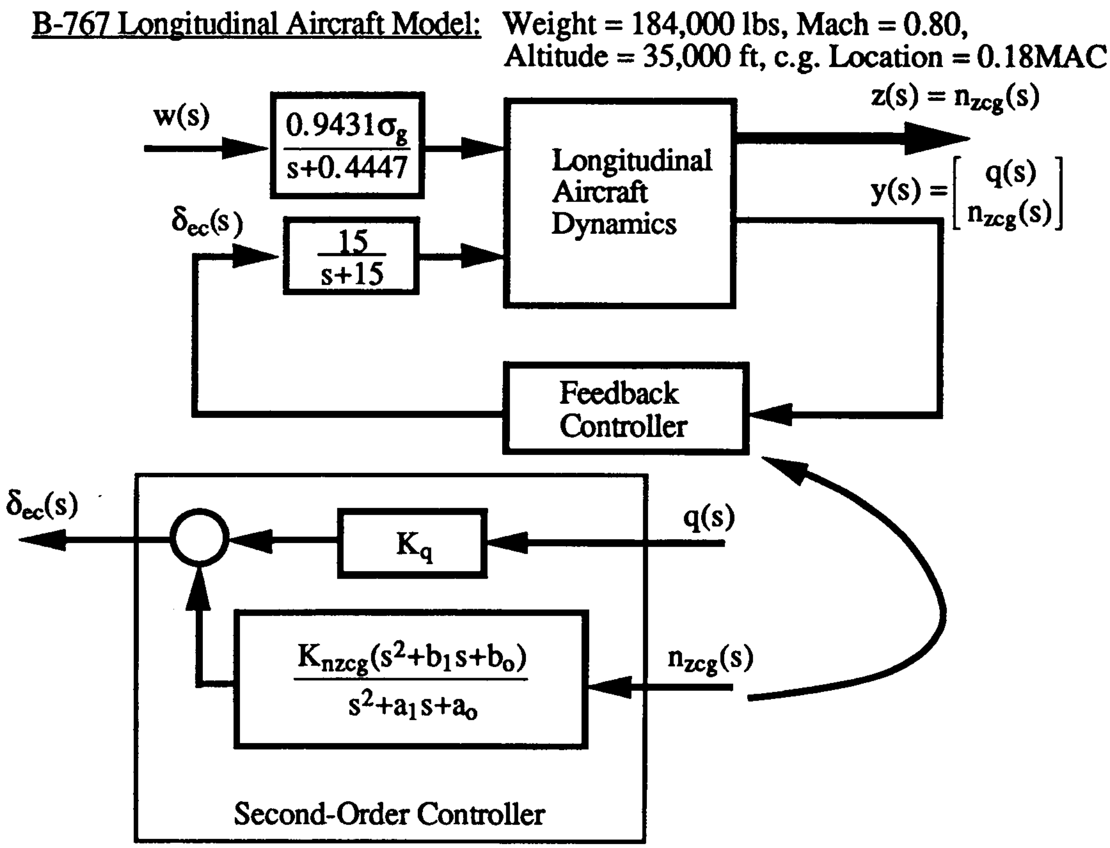

Figure 2 Disturbance Rejection Design for a B-767 Aircraft Using Mixed H^2 and H^∞-Optimization

Mixed H^2 and H^∞- performance objective is given by

$$J = \lim_{t_f \to \infty} \{ W_\infty \sup_{w(t)} \frac{\int_0^{t_f} [10 n_{zcg}^2(t) + \delta_{ec}^2(t)] dt}{\int_0^{t_f} w^2(t) dt} + W_2 E[10 n_{zcg}^2(t_f) + \delta_{ec}^2(t_f)] \} \quad (6)$$

where $\delta_{ec}(t)$ is the elevator control. State matrices of the design model are given in the appendix.

The controller is set up to have output feedback (Figure 2) on pitch rate q(t) and a second-order lead-lag feedback compensation on the normal acceleration $n_{zcg}(t)$. Figure 3 summarizes three controller designs illustrating trade-off achieved in mixed H^2 and H^∞-norm optimization:

(a) <u>H^2-norm optimization:</u> With W_2=1.0 and W_∞=0.0, this design simply solves the minimization of the mean square responses to Dryden turbulence using the controller structure shown in Figure 2.

(b) <u>Mixed H^2 and H^∞-norm optimization:</u> With W_2=1.0 and W_∞=1.0, this design is a predominantly H^∞-norm problem yielding results similar to those achieved with algorithms described in Reference 33. In this design the mean-square responses of $n_{zcg}(t)$ is roughly 16 percent higher than the H^2-norm optimized design (Case a) while the H^∞-norm is reduced by 20 percent. To recover the performance of H^2-optimized design (Case a), we simply increase the penalty W_2 on the H^2-norm performance.The following improvement is achieved with small degradation in H^∞-norm as seen in the next design case.

(c) <u>Improved mixed H^2 and H^∞-norm optimization:</u> With W_2=10.0 and W_∞=1.0, this design provides proper balance between H^2 and H^∞-norm performance. The resulting H^2-norm is almost the same as that of the H^2-optimized design (~ 1.3 percent higher) and the H^∞-norm is about 2.4 percent higher than that achieved in case (b).

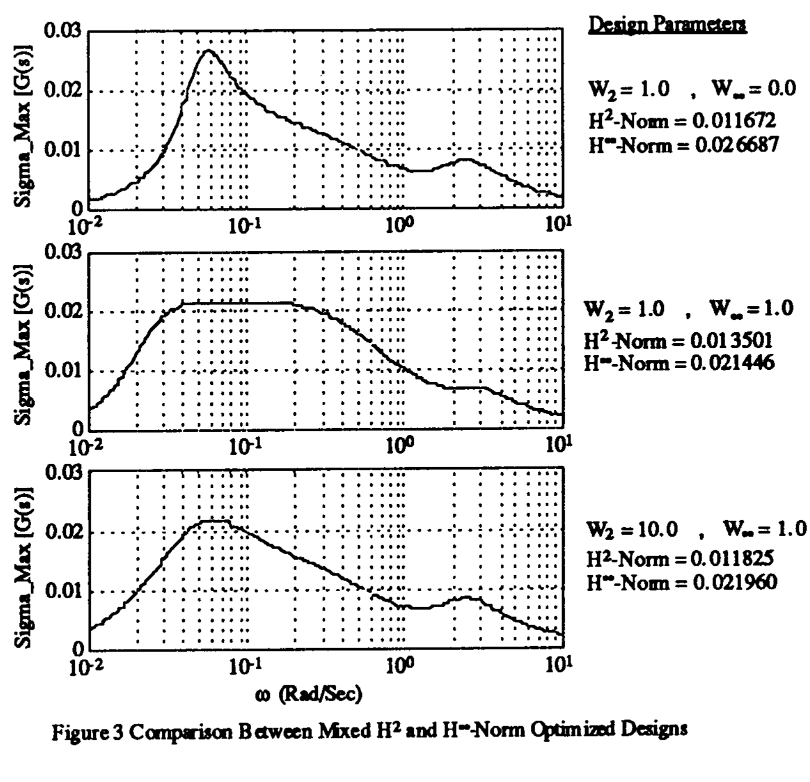

Figure 3 Comparison Between Mixed H^2 and H^∞-Norm Optimized Designs

Results of the H^∞-norm optimization are similar to those achieved by state-feedback or full-order output feedback designs using methods described in Reference 33 as indicated in Figure 4. Advantage of the current design approach is its simplicity and low order structure.

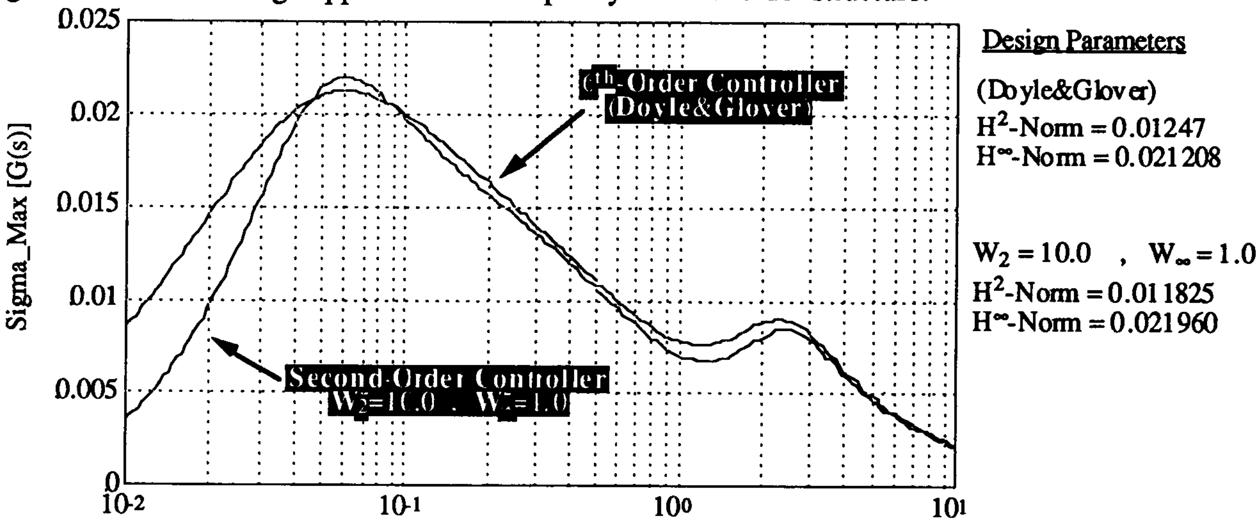

Figure 4 Comparison with Existing H^∞-Optimization Method

VI. Future Directions

Recent work have provided several useful mathematical measures for robustness characterization of multivariable feedback control design, numerical algorithms for their "exact" calculation and their usage in robust control-law synthesis. Basically there are two kinds of robustness measures depending on whether they are defined based on frequency-domain (i.e. Nyquist stability) or time-domain (i.e. Lyapunov stability) criteria.

Methods in frequency domain have made significant strive since the early work[12] initiated by Doyle. Analysis techniques to determine the μ-measure for structured uncertainty are still emerging and are most likely computationally intensive[13-18,20-21]. Complexity of these algorithms is partly the result of the wide variety of possibilities in the modeling of the uncertainty block Δ (Figure 5).

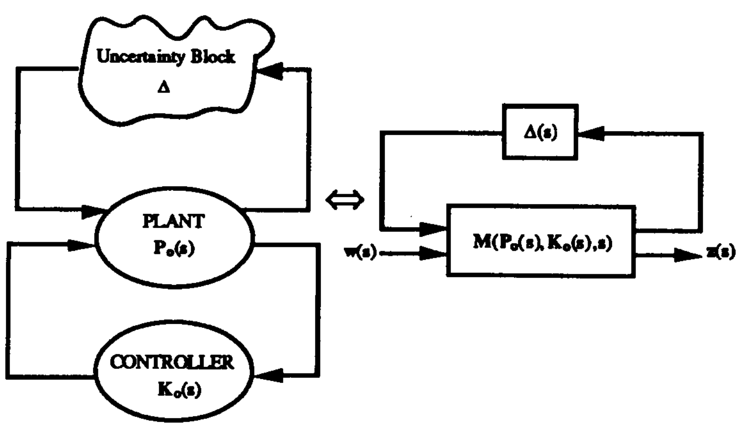

Figure 5 Robust Control Synthesis Based on Worst-Case Uncertainties of $\Delta(s)$

Generally, the more structure (i.e. information) one assigns to the uncertainty block Δ, the more difficult it is to determine the necessary and sufficient bounds on μ-measure[33].

One approach in robust control-law synthesis is to make use of recent methods for "exact" calculation of μ or related measures to define the worst-case uncertainty model, say Δ^*, and incorporate this condition into a closed-loop model $[I-\Delta^* M(s,K_0(s))]$ for re-optimization of the controller $K_0(s)$ embedded in the transfer matrix M(s).

For robustness measures based on time-domain approach[11], bounds on plant uncertainties ($\Delta A,\Delta B,\Delta C,\Delta D$) in the state matrices (A,B,C,D) are determined (Figure 6) providing sufficient conditions for closed-loop stability. These analysis procedures can be elaborated to obtain specific worst-case plant conditions. As before once established, these conditions can be implemented into the design procedure SANDY where one of the plant conditions represents the worst-case plant model from which appropriate design constraints for robustness can be defined.

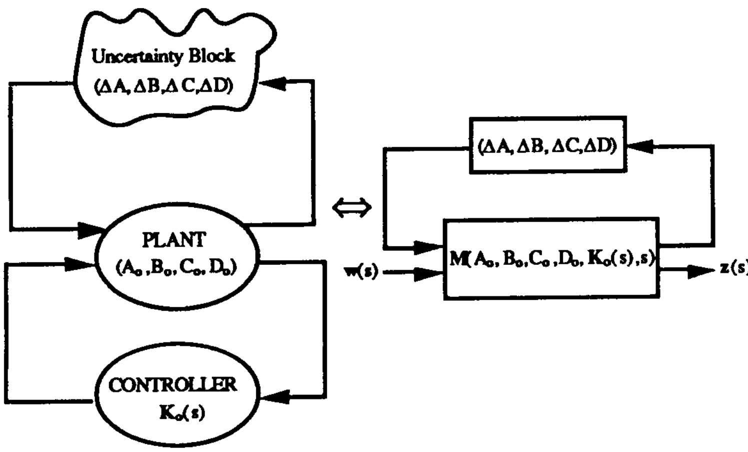

Figure 6 Robust Control Synthesis Based on Worst-Case Plant Perturbation ($\Delta A^*,\Delta B^*,\Delta C^*,\Delta D^*$)

Future work will also examine theoretical development of these robust design algorithms and their numerical implementation. These methods will be applied to the synthesis of flight control problems (e.g. SAS, autopilots, ride quality control, modal suppression, etc...) that include robustness issues such as multiloop stability margins, plant parameter variation and unmodelled high-frequency dynamics. Specifically we address the set-up of objective function and relevant design constraints (e.g. closed-loop damping, handling qualities in terms of short-period frequency, overshoots, command and control bandwidths, stability margins, limited control activities, etc...) for the following flight control problems,

(a) Stability Augmentation Systems:
- Pitch augmentation system
- Yaw damper design
- Disturbance rejection: ride quality, load factor reduction
- Structural mode stabilization: control of lightly damped structural modes

(b) Command Augmentation Systems:
- Integral Controls
- Autopilots: airspeed, altitude, flight path control
- Manual control with handling qualities
- Target tracking

As one might envision, the control synthesis depends strongly on the design objectives (e.g. inclusion of integral control, washout filters for decoupling in steady-state control, anti-aliasing filters, time delay, etc...) regardless of the methods used in the determination of feedback and feedforward control gains. One of our research goals is to identify components in the synthesis model essential to the design problems stated in (a) and (b) based on common knowledge of the design requirements in each particular situation.

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Appendix

The following are state matrices of the synthesis model with states $\{u,\alpha,q,\theta,x_w,\delta_e\}$, inputs $\{\delta_{ec},w\}$ and outputs $\{q,n_{zcg}\}$,

```
a =-1.6750e-02   1.1214e-01   2.8000e-04  -5.6083e-01   1.6750e-02  -2.4320e-02
   -1.6400e-02  -7.7705e-01   9.9453e-01   1.4700e-03   1.6400e-02  -6.3390e-02
   -4.1670e-02  -3.6595e+00  -9.5443e-01            0   4.1670e-02  -3.6942e+00
             0            0   1.0000e+00            0            0            0
             0            0            0            0  -4.4470e-01            0
             0            0            0            0            0  -1.5000e+01
b =          0            0
             0            0
             0            0
             0            0
             0   9.4310e-01
    1.5000e+01            0
c =          0            0   1.0000e+00            0            0            0
    6.9400e-03   3.2795e-01   2.3100e-03            0  -6.9400e-03   2.6790e-02
d =    0      0
       0      0
```